Response to the review

# Editor

*The paper has been revised by three reviewers and all agree that the work deserves publication. The reviewers, especially Reviewer 1, also reported some comments and suggestions to be addressed through a minor revision. The Authors could consider, as also suggested by one reviewer, to rephrase the title to better point out the theoretical nature of the work.*

Response: Thank you for providing us with the feedback from the reviewers. We are grateful for their thorough examination of our paper and their positive recommendation for publication. We acknowledge the constructive comments and suggestions, particularly those raised by Reviewer 1, and we will incorporate these changes in a revision to enhance the clarity and precision of our manuscript. We appreciate the suggestion to rephrase the title to better emphasize the theoretical nature of our work, and we will certainly consider this recommendation to ensure the alignment of the title with the content of the paper.

# Reviewer 1

*I miss the use of the distinctive symbol (^) on quantum operators with the intention of distinguishing them from variables that are not operators.*

Response: We appreciate the referee's note. In the text, as most variables are operators, we opted not to use the "^" symbols for a cleaner look. To distinguish average values from operators, we employ the bracket notation <…>. In the revised manuscript, we include a note clarifying our notation choice.

*Specifically, when discussing excitations in the waveguide since they are said to be considered as coherent states and treated classically. However, the text does not explicitly explain the reason for treating these states classically. My understanding is that the assumption involves a significant number of photons in the waveguide fields.* *In section II.E. Effective Hamiltonian, the waveguide field is said to be in a coherent state with amplitudes so I assume these are not operators, and later the incident power is constrained as , which makes me believe again that they are not operators.* *But then in the section III.A. Input/output formalism they are introduced as operators and their expectation values are taken.* *It maybe is a nomenclature thing, but it can appear confusing.*

Response: We thank the referee for the comments.

1. If one considers the driving field in the coherent state, the input field operator can be treated as a simple complex number due to the properties of coherent states [[1]](#footnote-1), [[2]](#footnote-2). Indeed, . Therefore, we do not need to consider the dynamics of the field operators and can use complex numbers to describe the field. Note that this approach is valid for any number of photons.
2. The referee may be confused between the degrees of freedom of the field, transmon operators, and output operators in the input/output formalism. Considering the scattering problem of the waveguide field and two transmon qubits, we still need to account for the qubits' degrees of freedom [[3]](#footnote-3). To describe the output of the system, we include variables describing the qubits, which, in our case, are operators. Thus, the output operator is the sum of the input field amplitude multiplied by a unity operator in the Hilbert space of two qubits and the transition operators of the qubits, see Eq. (21) in the text.
3. We agree with the referee that in Eq. (22), the brackets <…> should not be used for because as it was defined earlier, it is just a number, not an operator. We fix this provide an explanation in the revised version of the manuscript.

*Is there an assumption of Planck's constant as 1 when the anharmonicity is equated to the charging energy ? Otherwise, the units don't match.*

Response: Yes, the referee is correct. It was a typo; we missed the division by . This is rectified in the revised version of the manuscript.

*In certain references, the quantum master equation represents Γ as 2γ.*

Response: The referee is correct; notation varies among papers. In our manuscript, we follow the notation of Lalumiere [[4]](#footnote-4) and Muller [[5]](#footnote-5).

*The paper does not seem to present innovation but rather a theoretical explanation of the different parameters to consider when designing this system.* *The paper mainly describes the system implemented in reference [1].*

Response: We agree with the referee and have corrected the paper's title. The revised title is "Theory and analysis of nonreciprocity for superconducting qubits coupled to a waveguide."

*The state is not explained in the text but is mentioned several graphs.*

We thank referee for this comment. In the revised version of the manuscript, we add the definition of the and states.

*In the description of some figures, for example, Fig.2, the parenthesis indicating the specific picture is sometimes at the front of the description and sometimes at the end.* *The figures are referenced differently along the text. "2(a)", "Figs. 2(c-e)", "Figures 3(a,b)", etc*

Response: We thank the referee for this note. In the revised version of the manuscript, we adhere to the style of the IEEE T-MTT journal for consistent figure referencing.

*Page 3, title of subsection D says "Hamitonian".* *Page 9, left column, line 53, there is a lonely "w" I assume it's a "we".*

Response: We appreciate the referee's attention to detail. These typos are corrected in the revised version of the manuscript.

*It would be intriguing to observe, whether this proposed system can establish a quantum diode.*

Response: If the referee refers to the semiclassical system, it may indeed demonstrate quantum diode properties, but under unrealistic parameters. This situation arises because the system of two qubits coupled to the waveguide cannot be described semiclassically. For an adequate theoretical description, higher-order correlation functions are needed. We provide a detailed comparison between these approximations in section IV, in particular, see Fig. 9.

# Reviewer 2

*Great paper. Very thorough in the analysis. Minor comments:*

1. *You missed a \ in the γ in Subsection G.*
2. *Don't use the word "intricacies". Just say what you are including in a clear way.*
3. *Are the simulations closed form or from a circuit simulator? Its not clear.*

*The paper has absolutely no measurements. For T-MTT this is possibly a problem and in my view, is the main weakness. I would rename the title to include something like "Theory and analysis of …."*

*Best of luck in your research.*

We thank the referee for the high evaluation of our paper.

1. We appreciate the referee's observation. This typo is corrected in the revised version of the manuscript.
2. We agree with the referee and have rephrased the text for clarity in the revised version.
3. We thank the referee for this comment. We used the QuCAT library to obtain the parameters of the transmon circuits and then employed the quantum description of the transmons + waveguide system, as detailed in Eqs. (9-10) in the text. The revised manuscript includes this clarification.

# Reviewer 3

*This authors theoretically studied the nonreciprocal propagation in an one-dimensional waveguide consisting of two properly separated transmon quibits coupled to it. Specifically, the authors exploited the quantum correlation approach to model wave propagation properties and nonreciprocal effects in such a transmon qubit system, and derived the system's dynamics at the higher energy levels. This is very thorough general study of coupled transman qubits, shedding physical insights on the design and physical limits of two qubits quantum information processing systems, and may find many useful applications in the fields of quantum communication and computing. The paper is also very well written and organized, and the model is technically sound. The work presented herein is novel, timely, and of significant importance to the microwave society. Overall, I would recommend publication of this paper in IEEE Transactions on Microwave Theory and Technology.*

*I would also encourage the authors to study and/or comment the electrodynamics in two-qubits systems coupled directly inside a cavity-QED, such as a compact miniature waveguide, e.g., P. Y. Chen et al., "Miniaturization of Omnidirectional Cavity Antennas Using Substrate-Integrated Impedance Surfaces" IEEE Transactions on Antennas and Propagation, Vol. 69, 1728-1733 (2021); P. Y. Chen et al., "Waveguide Dispersion Tailoring by using Embedded Impedance Surface," Physical Review Applied, Vol. 10, 064024 (2018).*

We thank the referee for the evaluation of our work. We studied the papers the referee suggested and added the comments to the revised version of the manuscript.

1. Scully, M. O., & Zubairy, M. S. (1999). Quantum optics. [↑](#footnote-ref-1)
2. Carmichael, H. (1999). Statistical methods in quantum optics 1: master equations and Fokker-Planck equations (Vol. 1). Springer Science & Business Media. Chapter 2.3. [↑](#footnote-ref-2)
3. Carmichael, H. (1999). Statistical methods in quantum optics 1: master equations and Fokker-Planck equations (Vol. 1). Springer Science & Business Media. Chapter 2.3. [↑](#footnote-ref-3)
4. K. Lalumiere, B. C. Sanders, A. F. van Loo, A. Fedorov, A. Wallraff, and A. Blais, “Input-output theory for waveguide QED with an ensemble of inhomogeneous atoms,” Phys. Rev. A, vol. 88, no. 4, p. 043806, Oct. 2013. [Online]. Available: <https://link.aps.org/doi/10.1103/PhysRevA.88.043806> [↑](#footnote-ref-4)
5. C. Muller, J. Combes, A. R. Hamann, A. Fedorov, and T. M. Stace, “Nonreciprocal atomic scattering: A saturable, quantum Yagi-Uda antenna,” Phys. Rev. A, vol. 96, no. 5, p. 053817, Nov. 2017. [Online]. Available: <https://link.aps.org/doi/10.1103/PhysRevA.96.05381> [↑](#footnote-ref-5)